# WHAT CHIRAL SYMMETRY CAN TELL US ABOUT HADRON CORRELATORS IN MATTER

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The constraints imposed by chiral symmetry on hadron correlation functions in nuclear medium are discussed. It is shown that these constraints imply some structure of the in-medium hadron correlators, lead to the cancellation of the order $\rho m_\pi$ term in the in-medium nucleon correlator and result in the effect of mixing of the chiral partners correlators, reflecting the phenomena of partial restoration of chiral symmetry. The different scenarios of such restorations are briefly discussed.

## I. INTRODUCTION

There are little doubts that Chiral Symmetry (CS) is one of the most important principles of low-energy hadron physics. [1]. In the limit of massless quarks the QCD Lagrangian is symmetric with respect to the $SU(N) \times SU(N)$ chiral group. It is generally believed that this symmetry is spontaneously broken. CS breaking manifests itself in the absence of chiral multiplets of the particles with the same masses but different parities, for example, $\rho - a_1$ or $\sigma - \pi$ mesons. In the language of the correlation functions the broken chiral symmetry means that the lowest pole positions of vector and axial vector correlators, describing the $\rho$ and $a_1$ mesons , are different. The restoration of the symmetry in vacuum would result in the identity of the corresponding correlators which in turn leads to the same masses of the chiral partners. The other manifestation of the spontaneously broken CS is the

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occurrence of the nonzero order parameters like two quark condensate $\overline{q}q$. In the case of of hadron interactions in vacuum $q\overline{q}$=0 would imply that hadron masses become much smaller compared to its observed values. The relationships between condensates, hadron masses and corresponding correlators are significantly more complicated in the presence of medium. For example, the change of the nucleon mass in medium is not completely determined by the corresponding change of the quark condensate. The other example is the CS restoration for the correlators of the chiral partners. In matter the identity of the corresponding in-medium correlators does not nesseseraly means the degeneracy of the effective masses of chiral partners. The identity of the masses of chiral partners at the point of restoration is only one of the posibilities. The dynamics of hadrons in nuclear medium is described by the corresponding correlators. Let's consider the case of the two-point correlators. It can be written in the form

$$\Pi(p) = i\int d^4xe^{ip\cdot x}\langle\Psi|T\{J(x)J(0)\}|\Psi\rangle. \tag{1}$$

Where $J$ is the interpolating current in the corresponding hadron channel and the matrix element is taken over the ground states of the system with finite density $\rho$. The position of the lowest pole as the function of density determines the in-medium mass of hadron. If wave function $\Psi$ describes the infinite system of non interacting nucleons than the above correlator reflects the dynamics of the probe hadron moving in some mean field formed by nuclear matter. To calculate the corrections to this picture one needs to take into account the nuclear pions. Then the correlator can represented by the sum of two terms describing the contributions from the system of noninteracting nucleons and pionic corrections. Let's consider the specific part of this corrections where pion is emitted and then absorbed by nuclear matter. The corresponding piece of the correlators can be represented in the form

$$\int \frac{d\mathbf{k}}{4\omega_k}\frac{d\mathbf{k}'}{\omega_k'}\langle\Psi|a^{a\dagger}_{\mathbf{k}}a^b_{\mathbf{k}'}|\Psi\rangle\, i\int d^4xe^{ip\cdot x}\langle\Psi\,\pi^a(\mathbf{k})|T\{J(x)J(0)\}|\Psi\,\pi^b(\mathbf{k}')\rangle \tag{2}$$

The sum over the isospin indices is assumed. The terms with the different time orderings can be accounted for in a similar manner. We consider the nuclear pions in the chiral limit.

Since we are interested in the properties of hadron correlators which are related to chiral symmetry treating nuclear pions in the chiral limit seems to be a reasonable assumption in our case. By using the soft-pion theorem the part of the correlator describing the pionic corrections can be written as follows

$$\Pi^{\pi} = \frac{-i}{2}\xi \int d^4x e^{ip\cdot x} \langle\Psi|[Q_5^a, [Q_5^a, T\{J(x), J(0)\}]]|\Psi\rangle, \tag{3}$$

where we denoted $\xi = \frac{\rho\overline{\sigma}_{\pi N}}{f_\pi^2 m_\pi^2}$ and $\overline{\sigma}_{\pi N}$ is the leading nonanalytic part of the pion-nucleon sigma term $\sigma_{\pi N}$ The chiral expansion of $\sigma_{\pi N}$ reads as

$$\sigma_{\pi N} = Am_\pi^2 - \frac{9}{16}(\frac{g_{\pi N}}{2M_N})^2 m_\pi^3 + ....... \tag{4}$$

First, analytic term describes the short range contributions. In contrast, the nonanalytic term is due to long distance contribution of the pion cloud. Let's consider the correlator of two nucleon interpolating current. Matter can influence the properties of the QCD vacuum and thus change the two quark condensate. It is usually believed that the reduction of $< \overline{q}q >$ is related to the effect of partial restoration of CS. To the first order in the density the evolution of the two-quark condensate is given by [2,3]

$$\langle\Psi|\overline{q}q|\Psi\rangle = \langle 0|\overline{q}q|0\rangle(1 - \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2}\rho), \tag{5}$$

It turned out that not any change of $\overline{q}q$ is in the one-to-one correspondence with the CS restoration phenomena [4] since the terms of order $m_\pi$ presenting in the condensate are not allowed in in-medium nucleon mass. Let's show how to cancel the terms not allowed by CS in QCD sum rules. In QCD sum rules one relates the characteristics of QCD vacuum and the phenomenological in-medium nucleon spectral density. The effects of the low-momentum pions are long-ranged and stem from the phenomenological representation of the in-medium nucleon correlator. Assuming the Ioffe choice [6] of the nucleon interpolating current, making use of the transformation property of this current one can get the following chiral expansion of the in-medium nucleon correlator

$$\Pi(p) \simeq \Pi^0(p) - \frac{\xi}{2}\Big(\Pi^0(p) + \gamma_5\Pi^0(p)\gamma_5\Big), \tag{6}$$

Where $\Pi^0(p)$ is the nucleon correlator in chiral limit. It is useful to decompose the correlator into three terms with the different Dirac structures [3]

$$\Pi(p) = \Pi^{(s)}(p) + \Pi^{(p)}(p)\not{p} + \Pi^{(u)}(p)\not{u}, \tag{7}$$

where $u^\mu$ is a unit four-vector defining the rest-frame of nuclear system. Only the piece $\Pi^{(s)}(p)$ gets affected by the chiral corrections of order $\rho m_\pi$. Splitting the phenomenological expression of the nucleon correlator into pole and continuum parts one can get

$$\Pi(p) \simeq \Pi_{pole}(p) - \frac{\xi}{2}\gamma_5\Pi_{pole}(p)\gamma_5 + \left(1 - \frac{\xi}{2}\right)\Pi^0_{cont}(p) - \frac{\xi}{2}\gamma_5\Pi^0_{cont}(p)\gamma_5, \tag{8}$$

Where we denoted $\Pi_{pole}(p) \simeq (1 - \frac{\xi}{2})\Pi^0_{pole}(p)$ The explicit expression of the pole term has the following form [3]

$$\Pi_{pole}(p) = -\lambda^{*2}\frac{\not{p} + M^* + V\gamma_0}{2E(\mathbf{p})[p^0 - E(\mathbf{p})]}, \tag{9}$$

Here $M^*$ is the in-medium nucleon mass including the scalar part of the self energy and $\lambda^*$ is the nucleon coupling. Then one writes the three independent sum rules, one for each Dirac structure

$$-(1 - \frac{\xi}{2})\int dp\,\frac{w(p)}{2E[p^0 - E]} \simeq \frac{(1-\xi)}{\lambda^{*2}M^*}\int dp\,w(p)\left[\Pi^{(0,s)}_{OPE}(p) - \Pi^{(0,s)}_{cont}(p)\right] \tag{10}$$

$$-(1 + \frac{\xi}{2})\lambda^{*2}\int dp\,\frac{w(p)}{2E[p^0 - E]} \simeq \int dp\,w(p)\left[\Pi^{(0,p)}_{OPE}(p) - \Pi^{(0,p)}_{cont}(p)\right], \tag{11}$$

$$-(1 + \frac{\xi}{2})\lambda^{*2}V\int dp\,\frac{w(p)}{2E[p^0 - E]} \simeq \int dp\,w(p)\left[\Pi^{(0,u)}_{OPE}(p) - \Pi^{(0,u)}_{cont}(p)\right]. \tag{12}$$

Taking the ratio of these sum rules one can get the needed cancellation in the effective mass and vector self energy and bring the in-medium nucleon QCD sum rules in an agreement with the chiral symmetry constraints. Let's consider now the in-medium correlation function of the vector currents. The lowest pole of the isovector-vector correlator corresponds to the $\rho$-meson contribution. Due to relatively large width it decays inside nuclear interior so the spectrum of the produced dileptons can carry the information about the modifications of

the $\rho$-meson mass and width in matter. Such modification may be related with partial restoration of CS.One possible way to look at the phenomena of CS restoration is to study the correlators describing the in-medium dynamics of the chiral partners. The correlators of the chiral partners, in our case the correlators of the vector and axial-vector currents, should become identical in the chirally restored phase. Thus one can expect that these correlators get mixed when the symmetry is only partially restored. The effect of chiral mixing indeed takes place both at finite temperatures [7] and densities [8]. Making use of the standard commutation relation of current algebra $\left[Q_5^a, J_\nu^b\right] = i\epsilon^{abc} A_\nu^c$ and putting it in the expression for the correlator of the vector currents $\Pi_V$ one can get

$$\Pi_V = \Pi_V^0 + \xi(\Pi_V^0 - \Pi_A^0); \Pi_A = \Pi_A^0 + \xi(\Pi_A^0 - \Pi_V^0) \tag{13}$$

The parameter $\xi$ in this equation is 4/3 times the one for the nucleon correlator. $\Pi_V^0(\Pi_A^0)$ is the correlator of the vector (axial) currents calculated in the approximation of the non-interacting nucleons. As one can see from the above equations the correlators get mixed when soft pion contributions are taken into account. We note that this statement is model independent and follows solely from CS. CS implies that the correlators of the chiral partners acquire, due to mixing, the additional singularities. These singularities may manifest themselves in the, for example, dilepton spectrum, produced in the heavy-ion collisions. It means that the spectrum may show the additional enhancement at the energy region close to the mass of the $A_1$ meson, besides that at the mass of $\rho$ meson. The phenomena of mixing suggests few possible ways of how chiral symmetry restoration occurs. First, the lowest singularities of the both correlators could indeed coincide at the point of CS restoration. Second, the correlators may exhibit two poles of the same strength corresponding to the $\rho$ and $A_1$ mesons. Third, the width of the mesons could become large enough to make no longer sensible the whole concept of individual quantum state at high densities. One notes that the $\omega$ meson being an isospin singlet is not affected by the chiral corrections. It follows from the fact that the commutator of the isoscalar vector current with the axial charge $Q_a^5$ is zero. Let's briefly consider the mixing of the other type of chiral partners, namely the

$\sigma$-$\pi$ pair. Making use of the current algebra commutation relation $[Q_5^a, \pi^b] = -\delta^{ab} i\sigma$ and $[Q_5^a, \sigma] = -i\pi^a$ one can get

$$\Pi_\pi = (1+\xi)\Pi_\pi^0 - \xi\Pi_\sigma^0; \Pi_\sigma = (1+\xi)\Pi_\sigma^0 - \xi\Pi_\pi^0 \tag{14}$$

Here the mixing parameter is two times smaller than that for $\rho$-$A_1$ system and thus the effect of $\sigma$-$\pi$ mixing is less pronounced at the normal nuclear density than in the case of $\rho$-$A_1$ mixing. The point of the complete restoration of chiral symmetry should, of course, be the same for all kinds of the chiral partners but the "velocity" of approaching to this point may well be different. Similar to the case of the $\rho$-$A_1$ system the mixing in the pseudoscalar-scalar channel practically means that the correlators exhibit the singularities which are dictated by CS and should be taken into account regardless of the model used to describe the concrete hadronic processes. However, the effect of the $\sigma$-$\pi$ mixing can probably be observed at relatively large densities. The case worth studying is deeply bound pion states [9] in heavy nuclei.

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## REFERENCES

[1] C.Adami and G. E. Brown, Phys. Rep. **224** (1993) 1;

[2] E. G. Drukarev and E. M. Levin, Nucl. Phys. **A511** (1990) 679.

[3] T. D. Cohen, R. J. Furnstahl and D. K. Griegel, Phys. Rev. Lett. **67** (1991) 961.

[4] M. C. Birse, Phys. Rev. **C53** (1996) 2048

[5] S. Weinberg, Phys. Lett. **251** (1990) 288; Nucl. Phys. **B363** (1991) 1; Phys. Lett. **B295** (1992) 114.

[6] B. L. Ioffe, Nucl. Phys. **B188** (1981) 317; **B191** 591(E).

[7] M.Dey, V.L.Eletsky and B.L.Ioffe, Phys. Lett. **B252** (1990) 620.

[8] B. V. Krippa, Phys. Lett. **B427** (1998) 13.

[9] M. V. Kazarnovski, B. V. Krippa , Phys. At. Nucl. 56 (1993) 5.